Promoting Collaboration in Courses with Perceived Single Correct Solutions

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Abstract

With the role of the Internet rapidly increasing in higher education, teachers must search for ways to create meaningful learning experiences for their students. The use of traditional discussion boards to facilitate collaborative problem solving in science and mathematics courses can be problematic with a tendency for discussions to stop once a perceived correct solution has been posted. This paper presents two alternative approaches to collaborative problem-solving that may be more appropriate in such cases. Both methods of facilitation can be called "shared-work" approaches in which students work privately before their work is shared with classmates. One approach, developed by Thomas Banchoff, is an individualized approach in which each student is responsible for constructing his or her own solutions to an assignment in a personal space that is shared with the entire class during a secondary phase. During the secondary phase, the students can add to and finalize their responses while having access to the work of others. A small-groups version of this approach was developed by the author to promote more student-to-student interactions by requiring the students to work in small groups during the second phase.

Keywords: online instruction, problem-solving, collaborative learning, classroom management, course design, delivery methods

Introduction

A common suggestion for getting students involved in their own learning is to have the students work together in small groups on assignments. There is considerable research evidence that the use of small groups is beneficial to students in undergraduate science, technology, engineering, and mathematics (STEM) courses. Springer, Stanne, and Donovan (1999) conducted a meta-analysis of studies pitting the use of small-groups against individualized approaches, and they reported that the students working in small-groups tended to have higher levels of achievement and persistence, as well as more positive attitudes toward the courses. Davidson, Reynolds, and Rogers (2001) argue for the use of collaborative groups in undergraduate mathematics courses, noting several benefits. A general benefit of collaboration is the development of interpersonal skills required to work successfully within a group. More specifically, Davidson, Reynolds, and Rogers argue that students can learn from the different perspectives provided by their group members, and the process of explaining ideas and concepts to others often improves one's own understanding. Furthermore, the requirement that students work in groups for mathematical problem solving might promote the view of mathematics as a human activity and challenge beliefs that all mathematical problems have single correct solutions that can be obtained by routine procedures.

Potential Difficulties with Asynchronous Collaborative Problem Solving

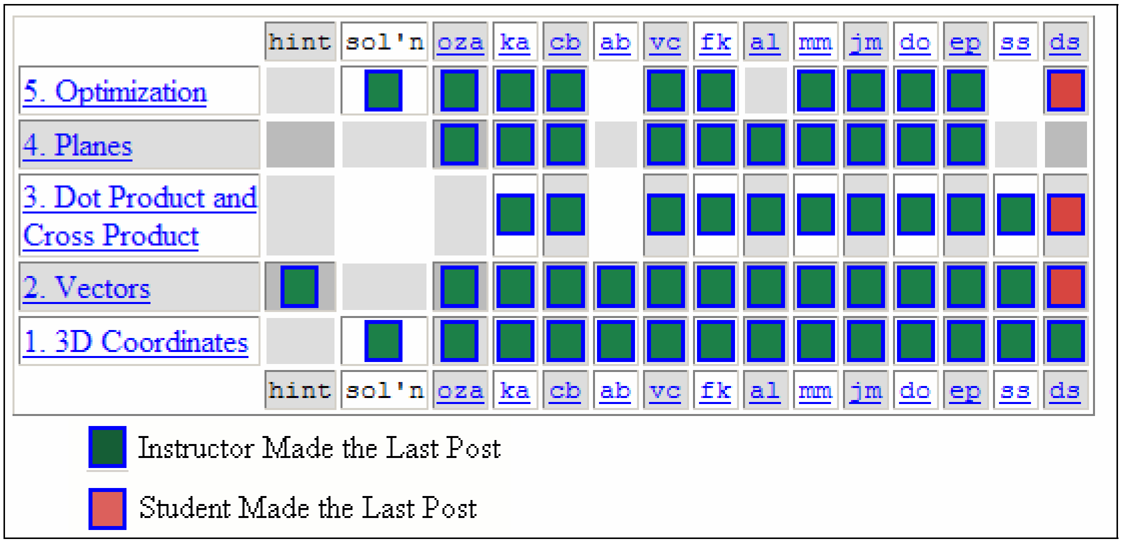
While there are many challenges to successful face-to-face collaborative problem solving, a separation of time and space creates a new set of difficulties for using group activities in an online environment. There are tools that allow students to communicate by audio, video, text, or multimedia in real time, but one argued advantage of online education is a greater amount of freedom for students to work on their own schedules. Therefore, it is important to investigate ways to facilitate problem solving in an asynchronous setting. For the purpose of this paper, the term asynchronous refers to any form of communication that does not occur in real time.

There is an extensive research base documenting obstacles to meaningful participation in asynchronous assignments. Harasim (1986) cites a loss of visual cues, the difficulty and time needed to reach a group consensus online, health problems such as eyestrain and backstrain, and a variety of problems related to technology as disadvantages of online instruction. Bullen (1998) investigated student perceptions of factors influencing participation in an online computer science ethics course and developed a conceptual framework with four categories: attributes of the medium, design of the learning activities, student dispositional factors, and student situational factors. Bullen identified many potential obstacles to online participation in each of these categories. The purpose of this paper is to discuss a particular difficulty that arises in online problem solving when there is a perceived "correct" answer and to address a potential approach to combating this difficulty.

A common tool for facilitating asynchronous communication over the Internet has been a discussion board. With traditional discussion boards, once a message is created, it is stored by an online server and retrievable by anyone with access to the discussion board. While it is easy to imagine such a communication tool facilitating a lively discussion or debate in a humanities or social science course, it seems more problematic to use a discussion board for collaborative problem solving in a STEM course. Although many educators recommend the use of open-ended problems with no clearly correct solution or multiple correct solutions, many problems used in STEM classes are closed ended with single correct answers. Such problems may be ill suited for use with traditional discussion boards. Instead of proposing ideas, asking questions, or starting general discussions, students in the author's College Algebra classes have tended to post proposed solutions. Once a solution that is perceived to be correct has been posted, discussion tends to stop or becomes limited to statements of agreement. This type of behavior leads to two problems commonly found with any form of group work. Either a small number of students dominate the discussions without providing opportunities for others, or certain group members willingly allow other students to do all of the work without contributing. One potential solution to this problem is a two-phase approach such as that used by Thomas Banchoff (2005) with his "communication tensor."

Banchoff's Communication Tensor

Thomas Banchoff (2005) uses a course management system that he developed, especially for mathematics courses, with his students at Brown University. The main tool for facilitating student work in Banchoff's software is called the communication tensor or the tensor for short. Banchoff calls it a tensor because student work is stored and displayed in a multidimensional array. When a user accesses the main webpage for the tensor, he or she sees a two-dimensional array with a row for each assignment and a column for each student. By clicking on an assignment name, the user is taken to a webpage with another array broken down by individual problems. Figure 1 shows the instructor's view of the tensor in a class with 5 assignments and 13 students. The color codes indicate whether the instructor or a student made the last post in an assignment space.



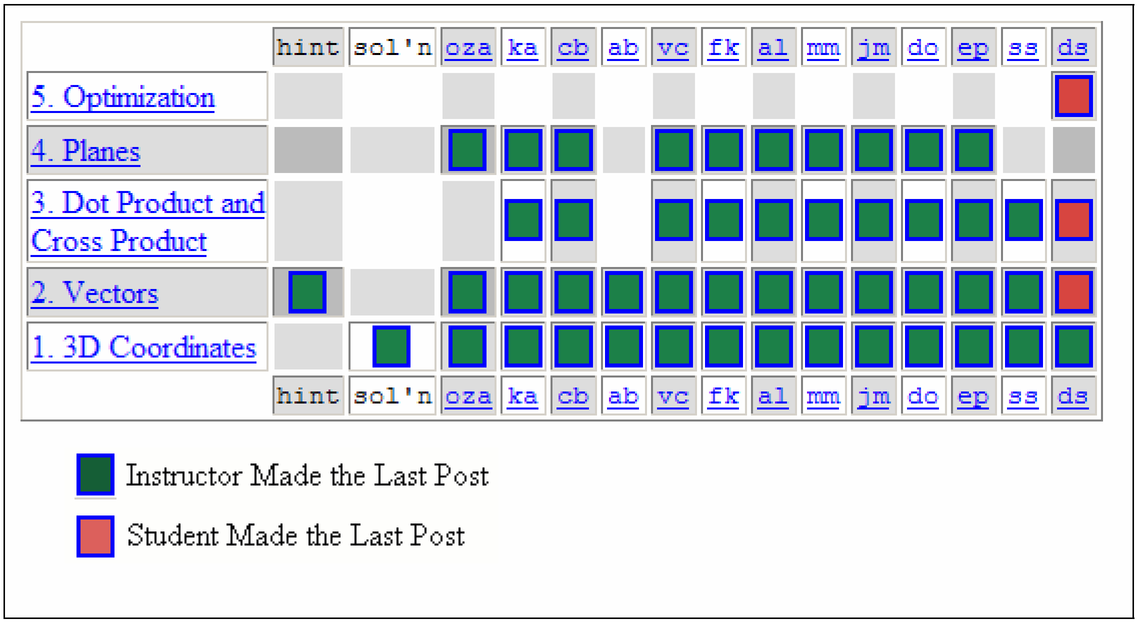
	hint	sol'n	oza	ka	cb	ab	vc	fk	al	mm	jm	do	ep	ss	ds
5. Optimization		🟩	🟩	🟩	🟩		🟩	🟩		🟩	🟩	🟩	🟩		🟥
4. Planes			🟩	🟩	🟩		🟩	🟩	🟩	🟩	🟩	🟩	🟩		
3. Dot Product and Cross Product				🟩	🟩		🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟥
2. Vectors	🟩		🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟥
1. 3D Coordinates		🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩
	hint	sol'n	oza	ka	cb	ab	vc	fk	al	mm	jm	do	ep	ss	ds

🟩 Instructor Made the Last Post

🟥 Student Made the Last Post

Figure 1. An instructor's view of the assignment array in Banchoff's communication tensor for a class with 13 students and five assignments. Blank spaces indicate that no posts have been made.

While the row by column format of organizing the students' work is a novel approach, the real innovation of Banchoff's communication tensor is the way that assignments are divided into individual and whole class phases. When an assignment is written using Banchoff's software, the instructor is given the option to set a timed lock that controls when access will be granted to the whole class. If an assignment is locked, each student's work is only accessible to that student and the instructor. Once an assignment is unlocked, any user can access any student's work by clicking on a colored square in the tensor. There is a feature that allows the students to create private posts that can only be viewed by the student who makes the post and the instructor at all times, but the students are encouraged to make public posts and share as much as possible. Figure 2 shows a sample screenshot from the perspective of a student with user name *ds*. Because the assignment labeled "Optimization" is locked for the initial phase, the student cannot see hyperlinks to his classmates' work. Each of the other assignments is unlocked, and the student can see a hyperlink for each student who has attempted the problems.



	hint	sol'n	oza	ka	cb	ab	vc	fk	al	mm	jm	do	ep	ss	ds
5. Optimization															🟥
4. Planes			🟩	🟩	🟩		🟩	🟩	🟩	🟩	🟩	🟩	🟩		
3. Dot Product and Cross Product				🟩	🟩		🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟥
2. Vectors	🟩		🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟥
1. 3D Coordinates		🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩
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🟩 Instructor Made the Last Post

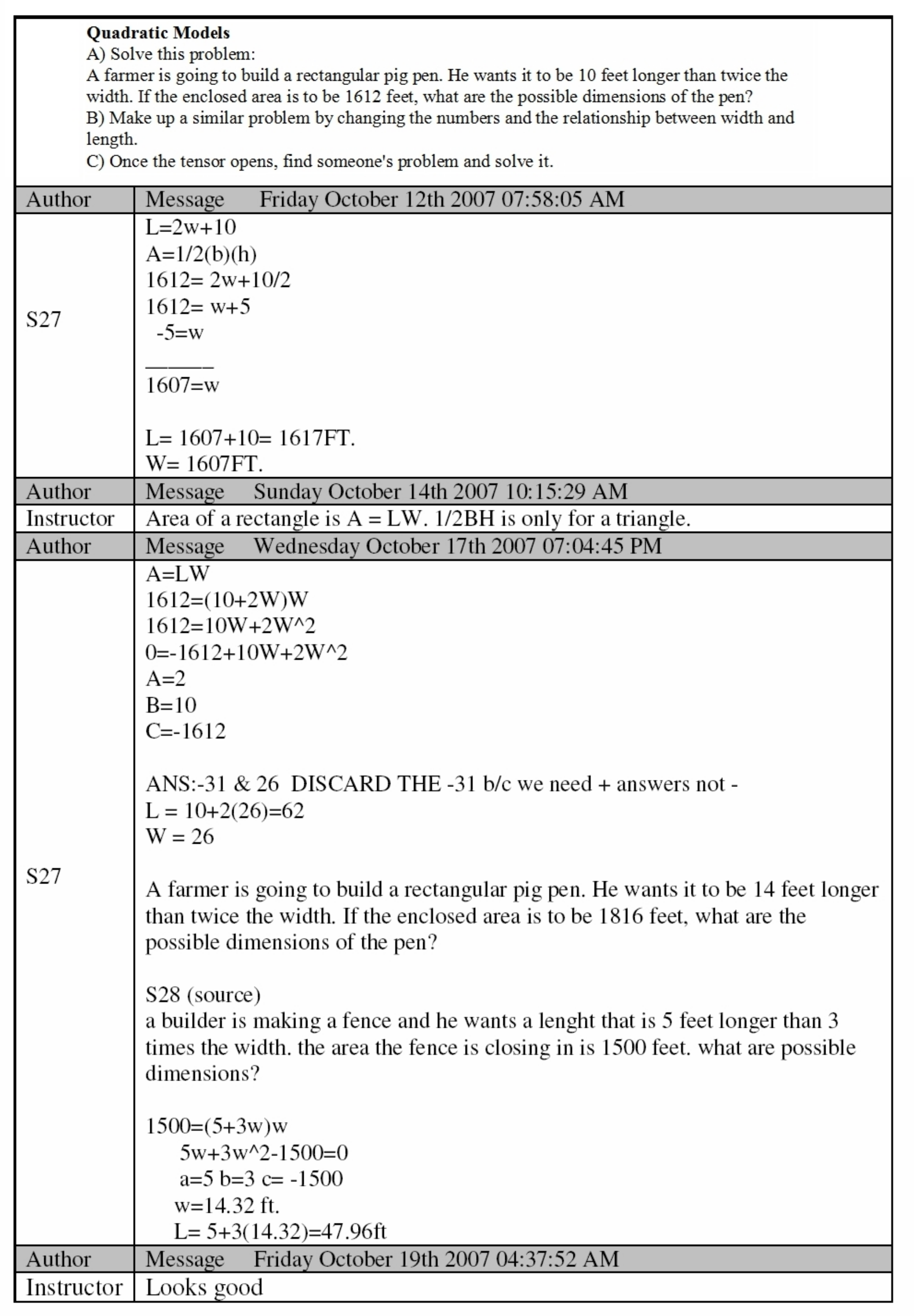
🟥 Student Made the Last Post

Figure 2. The view of the assignment array in Banchoff's communication tensor for a class with five assignments and 13 students from the perspective of a student with the username ds. The assignment labeled Optimization is locked and the other assignments are unlocked.

The Shared-Work Approach

The method of facilitating online problem solving used by Banchoff with his communication tensor can be called a "shared-work approach" because each student is responsible for completing his or her own assignment in an individual space, and the students' work is shared through the tensor. The students are not assigned to specific groups and are not explicitly required to collaborate, but they have access to the shared work to use as they desire. With the individual initial phase, the shared-work approach prevents some of the common problems associated with group work such as one or more students dominating discussions or students relying on group members to do all of the work.

Figure 3 provides an example of an assignment on quadratic modeling from one of the author's College Algebra classes and the messages posted by a student with pseudonym S27 and the author in the student's assignment space. The first two messages were posted during the initial phase when S27 only had access to her own space. In her first attempt at the problem, S27 started with the area formula for a triangle instead of a rectangle, and her algebra was incorrect. Trying to give just enough help to keep her working toward a correct solution, the author made a comment on the area formulas. S27 did not make another post on this assignment until the second phase, when the tensor was open. She then correctly solved the initial problem, posed her own problem, and solved a classmate's problem that she had found in the tensor. It is worth noting that student S28 had made up a correct problem, but he had no correct work toward a solution to his problem or the original problem.



Quadratic Models
A) Solve this problem:
A farmer is going to build a rectangular pig pen. He wants it to be 10 feet longer than twice the width. If the enclosed area is to be 1612 feet, what are the possible dimensions of the pen?
B) Make up a similar problem by changing the numbers and the relationship between width and length.
C) Once the tensor opens, find someone's problem and solve it.

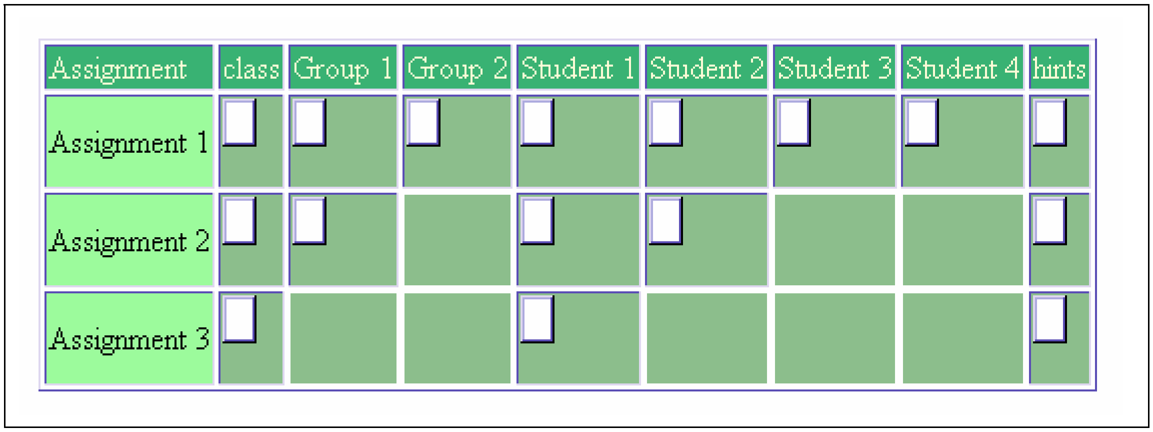
Author	Message Friday October 12th 2007 07:58:05 AM
S27	L=2w+10 A=1/2(b)(h) 1612= 2w+10/2 1612= w+5 -5=w ______ 1607=w L= 1607+10= 1617FT. W= 1607FT.
Author	Message Sunday October 14th 2007 10:15:29 AM
Instructor	Area of a rectangle is A = LW. 1/2BH is only for a triangle.
Author	Message Wednesday October 17th 2007 07:04:45 PM
S27	A=LW 1612=(10+2W)W 1612=10W+2W^2 0=-1612+10W+2W^2 A=2 B=10 C=-1612 ANS:-31 & 26 DISCARD THE -31 b/c we need + answers not - L = 10+2(26)=62 W = 26 A farmer is going to build a rectangular pig pen. He wants it to be 14 feet longer than twice the width. If the enclosed area is to be 1816 feet, what are the possible dimensions of the pen? S28 (source) a builder is making a fence and he wants a lenght that is 5 feet longer than 3 times the width. the area the fence is closing in is 1500 feet. what are possible dimensions? 1500=(5+3w)w 5w+3w^2-1500=0 a=5 b=3 c= -1500 w=14.32 ft. L= 5+3(14.32)=47.96ft
Author	Message Friday October 19th 2007 04:37:52 AM
Instructor	Looks good

Figure 3. The content of student S27's assignment space for an assignment on quadratic modeling used with a communication tensor and a shared-work approach.

A Small-Groups Version of the Shared-Work Approach

Banchoff's tensor has the advantage of requiring individual contributions while providing opportunities for collaboration. However, based on the author's experiences as a student and a teacher using the tensor, the student-to-student interactions can be limited when compared to traditional group work. With the use of individual assignments, a student's goal becomes to solve the assigned problems in his or her own assignment space. With time constraints and many other commitments, students tend to look at the other students work only when they perceive that it is necessary. When the students in a section of College Algebra taught by the author using an individualized shared-work approach were asked how they had found the tensor helpful (Cooper, 2008), the most popular responses were that they would look for help whenever they were stuck or that they would look at other students' work to check their own. When asked why they did not look at the work of others more often, several students said that they felt no need to do so when they were confident in their own work.

In order to promote more student-to-student interactions while maintaining the advantages of an individual initial phase, the author has implemented a small-groups version of the shared-work approach, using a version of the tensor with an extra lock for facilitating group work. In the author's software, when an assignment is locked at a group level, each member of a group can access the work of his or her group members. This allows an initial phase where students work alone and a second phase in which they continue to work within small groups. Figure 4 displays a sample screenshot from the author's software from a demonstration class with 4 students and 2 groups. This screenshot is from the perspective of Student 1, who belongs to Group 1 with Student 2. Students 3 and 4 make up Group 2. Assignment 3 is locked at an initial phase, and Student 1 can only access his own space and areas for hints and comments for the whole class. Assignment 2 has progressed to a secondary group phase in which Student 1 can access his own space as well as Student 2's space and their group space. Assignment 1 is unlocked at the class level, an option that instructors can use to make all work on an assignment available to the entire class.



Assignment	class	Group 1	Group 2	Student 1	Student 2	Student 3	Student 4	hints
Assignment 1	☐	☐	☐	☐	☐	☐	☐	☐
Assignment 2	☐	☐		☐	☐			☐
Assignment 3	☐			☐				☐

Figure 4. Student 1's view of the tensor in a small-groups section using the author's communication tensor. Assignment 1 is unlocked at the class level; Assignment 2 is locked at the class level but unlocked for groups; and Assignment 3 is locked for an individual phase.

The tensor structure is one way to accomplish a small-groups version of the shared-work approach, but instructors can use any software that allows individual submissions (such as e-mail) and small-group discussions. Figure 5 contains an outline of the two versions of the shared-work approach discussed in this article.

Student Assignments

While the term *problem* is often used in mathematics to describe any exercise, O'Daffer, Charles, Cooney, Dossey, and Schielack (2005) describe a problem as a situation that involves a question that is challenging to the students and that cannot be answered immediately by some routine procedure known to the students. The author has attempted to use problems in online assignments that satisfy these criteria. In addition to presenting non-routine challenges, problems with multiple correct solutions or solution strategies are even better. For example, a routine exercise would be to find an equation for a line passing through two given points. A more challenging open-ended assignment is to find equations of three lines that intersect to form a triangle with a given area. In a shared-work approach, students are given time to grapple with such a question on their own, and then they are given access to the work of the whole class or a small group. At the end of the individual phase, it is likely that several different correct specific solutions have been found and that several students have been unable to find a solution. During the shared-work phase, the students are exposed to different solutions. The students who were unable to solve a problem can observe the strategies used by their classmates and create their own solutions. Ideally, the students will discuss general solution strategies and properties of the assignment. With this type of assignment, the students get to experience a problem-solving situation and practice basic skills. By the time that a student has developed equations for a triangle with a given area, he or she has practiced finding an equation of a line through given points, likely including horizontal and vertical lines. By using a non-routine situation the students are given an opportunity to learn that struggling with an assignment is acceptable and that mathematics is not simply about memorizing rules and procedures.

The Shared-Work Approach

Initial Phase

During an initial phase, the students are assigned a set of problems and given access to an online space to construct their individual answers and interact with the instructor. During this phase, the students cannot access any of their classmates' work on the assignment.

Second Phase- Individual Approach

During a second phase, the students are responsible for finalizing their responses to the problems. Each individual student is responsible for the answers constructed in his or her own assignment space while having access to each classmate's work.

Second Phase- Small-Groups Approach

During a second phase, the students are responsible for completing the assignments in small groups (3 or 4 students). Each student has access to a group space and all of the work created during the initial phase by the members of the group.

Figure 5. Outline of the individual and small-group versions of the shared-work approach to online problem solving.

To date, the author has used the small-groups approach and the individualized shared-work approach with the same types of assignments. In an ideal situation, students working under the individualized approach would use the whole-class phase to seek out the work of classmates to compare and contrast solutions and to discuss the problems in detail. It has been the author's experience that having access to the work of every student in a class can be overwhelming. Many students do not have the time or patience to look at the work of every other student. By using a small-groups variation of the shared-work approach, the students still have individual responsibility, and they are able to focus on the work of a small number of classmates. It is also likely that identifying oneself with a group of three or four classmates will create a stronger sense of responsibility to the group. Through experimentation with the two approaches to online facilitation, the author has found higher levels of participation and more direct student-to-student interaction when the students are placed in small-groups (Cooper, 2008).

Discussion

This paper offers two possible alternatives to traditional collaboration for use with online problem solving. The two approaches incorporate a two-phase method of facilitation to promote individual accountability and collaboration. The individualized approach may be appealing to many instructors. Using the individualized shared-work approach, the students have the potential to learn from the work of their classmates, or they may choose to work independently, interacting with only the instructor. With the increasing importance of technology, one could argue that the students need to be able to decide when and where to seek information as needed. Deciding what to look at and how to use what they find in the tensor may help students develop these types of skills.

The small-groups version of a shared-work approach maintains individual accountability, but provides the students with structured groups. Instead of sorting through the contents of dozens of individual student assignment spaces, the students can focus on working with their small group. The small-groups approach should also be easier to adapt to course management systems, such as WebCT, that do not include a communication tensor. An instructor could simply set up group discussion areas and require each student to provide initial responses by email prior to working in groups.

There are many problems recognized in distance education that neither of these approaches eliminates. Mathematical problem solving typically involves the use of symbols and drawings or graphs. Entering items other than standard text into communication software can be difficult and time consuming. Some students might have difficulty accessing the required technology, and some students could require extra help learning how to use the technology. In addition, it has been reported that many students view mathematics as a subject that one does alone with pencil and paper and that mathematics is about using memorized procedures to find numerical solutions (Schoenfeld, 1989; Schommer, Crouse, & Rhodes, 1992). It is difficult to get students to interact and discuss mathematics if they believe that the only purpose is to find the single correct answer, but teachers must work to find ways to push students toward the development of meaningful understanding. The approaches to online problem solving described in this paper are ways that the author has used to present challenging problems in undergraduate mathematics classes. Teachers and researchers are encouraged to explore these approaches and to continue searching for better tools and methods of promoting online problem solving.

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